International Journal of Advanced Research in Physical Science (IJARPS)
Volume 2, Issue 7, July 2015, PP 1-18
ISSN 2349-7874 (Print) & ISSN 2349-7882 (Online)
www.arcjournals.org

Einstein was Correct: A Deeper Level of Quantum Theory is Needed

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Abstract: *It appears now that Eintstein's original proposal that quantum mechanics should not be considered as a complete description of physical reality was correct. An argument put forth here for this conjecture is based on the recent publication that the unusual spin correlation of singlet state pair particles can be better understood using a more complete and complex nonlinear model than is contained in traditional quantum mechanics. It is shown that similar arguments for understanding the spin correlation of entangled particles could have been utilized by Einstein in his original EPR publication in order to exonerate his proposal that the quantum mechanical description of physical reality is not complete. Although the original EPR publication fell short of presenting a convincing argument that there is evidence for a deeper level of reality than is found in the quantum mechanical wave function description, it is proposed here that their intent was correct. First, the original EPR argument associated with non-commuting position and momentum operators, as well as the equivalent Bohm argument associated with non-commuting orthogonal spin operators, are reviewed. However, with the advent of the Bell inequality argument, associated with general spin measurements of two separated singlet state pair particles, the discussion of a deeper level of local quantum reality was side-tracked due to the general consensus that a nonlocal spin correlation was responsible for the unusual spin measurement results. Finally, with consideration of the recent publication that the spin correlation results can be explained using a more complex spin model, it is proposed that Einstein was correct all along, since there is finally evidence that the quantum mechanical wave function description of reality is not complete.*

Keywords: *EPR Publication, Bohm Spin Correlation, Bell Inequality, Quantum Foundations, Deterministic Chaos.*

1. INTRODUCTION

It has recently been published that it may be possible to understand the unusual quantum mechanical spin correlation of singlet state pair particles in a similar fashion that one understands classical deterministic chaos found in nonlinear dissipative systems [1]. With this in mind, the original EPR publication by Einstein [2] is revisited in order to exonerate the proposal that the quantum mechanical description of reality is not complete. Unfortunately the original EPR publication fell short of presenting a convincing argument that there is evidence for a deeper level of reality than is found in the quantum mechanical wave function description; however, it will be shown in the following that their intent to demonstrate the incompleteness of quantum mechanics was profound.

In section 2, the shortcomings of the original EPR argument will be addressed, which is associated with the proposal that there is a simultaneous reality of the momentum and the position of two separated but precisely anti-correlated particles. Here, the reader will be reminded that in quantum systems, where the momentum operator, $\hat{p}$, and the canonically conjugate position operator, $\hat{q}$, do not commute, simultaneous measurements of both non-commuting variables is not possible. In section 3, the equivalent shortcomings of the Bohm correlated spin measurement proposal [3] will be addressed, which explores the simultaneous reality of two orthogonal spin values of two separated but precisely anti-correlated singlet state particles. Again, the reader will be reminded that in quantum systems, where the z component of the spin operator, $\hat{\sigma}_z$, and the orthogonal x component of the spin operator, $\hat{\sigma}_x$, do not commute, simultaneous measurements of both non-commuting variables is not possible. In section 4, the generalization of the EPR/Bohm spin correlation analysis of two separated singlet state particles in arbitrary spin measurement directions will be addressed, which was originally used in the Bell inequality argument [4]. Here, the reader will be reminded that the unusual correlation between spin measurements of the two separated singlet state particles led much of the physics community to assume that there was a nonlocal process involved which led to thespin correlation results [5,6]. In section 5, an alternative understanding of the spin correlation results will be addressed [1], which relies on the use of a more complex local spin model than is used in traditional quantum mechanics. Here, the reader will be reminded of similar non-predictable emergent behavior which is found in classical nonlinear dissipative systems which exhibit deterministic chaos [7]. In section 6, a discussion will be given of the application of more complex quantum models to address the EPR argument, which go beyond traditional linear operator theory that have previously incorporated the mystical notion of a wave function collapse. Finally, in section 7, conclusions will be given which are associated with vindication of Einstein's proposal that quantum mechanics should not be considered as a complete description of physical reality.

2. Brief Review of the EPR Argument

The EPR argument [2], put forth in 1935, essentially addressed a two-particle wave function, $\psi(y_1, y_2)$, which was constructed using an infinite superposition (a Fourier integral representation) of two different orthonormal basis eigenfunctions of non-commuting operators (the momentum and the position operators).It is assumed that two identical particles travel in opposite directions, with opposite momenta, away from each other along the y axis of separation, as shown in Fig. 1. The two-particle wave function was constructed to represent the entangled position (y_1 for the first particle, labelled as 1, and y_2 for the second particle, labelled as 2) and the canonically conjugate momenta (p_1 for the first particle, and p_2 for the second particle) of two precisely anti-correlated particles, which separate in opposite directions, where classically the separation distance is $y_1 - y_2 = \Delta y$, with opposite momenta, where $p_2 = -p_1$, due to an initial impulsive force which occurred when both particles, each with zero momenta, $p_1 = p_2 = 0$, were at the origin, $y_1 = y_2 = 0$. The objective of the argument was to consider measurements on the first particle, without disturbing the second particle, to show that two non-commuting elements of reality for the second particle (the momentum and the position) could be known precisely and simultaneously. However, as this is not possible for a quantum mechanical wave function description of reality, which must satisfy the uncertainty principle, it was then concluded that the wave function description of quantum mechanics is not complete, as it would need to include this additional information about these precise elements of reality for two non-commuting operator measurements.

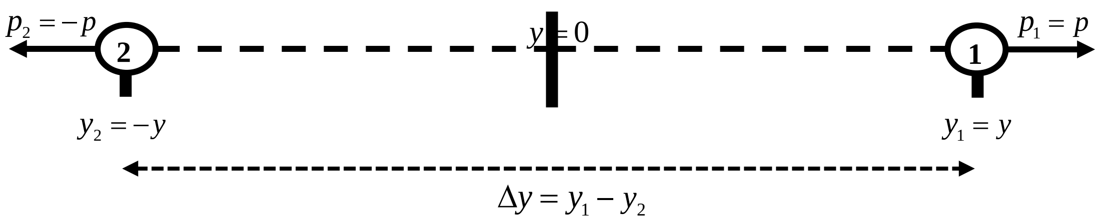

Fig1. *EPR model of two particles traveling in opposite directions.*

To demonstrate the details of this argument, it is useful to recall that free particle momentum eigenfunctions, $\varphi_k(y) = \frac{1}{\sqrt{2\pi}} e^{iky}$, can be labelled using the wavenumber index, k, where the momentum eigenvalue is $p = \hbar k$, the normalized Planck constant (the Planck constant, h, divided by 2π) is $\hbar = h/2\pi$, and the momentum operator with respect to the y coordinate is $\hat{p} = -i\hbar\partial/\partial y$ .Also note that this is a solution of the free particle Schrödinger equation for a particle of mass m, where the energy eigenvalue is $E = (\hbar k)^2/2m$, and the particle has equal probability of being anywhere along the y axis.Using these momentum eigenfunctions, it is useful to combine an eigenfunction for the first particle, $\varphi_k(y_1)$, with momentum of p, with an eigenfunction of the second particle, $\varphi_{-k}(y_2)$, with an opposite momentum of $-p$, using a direct product, as $\varphi_k(y_1)\varphi_{-k}(y_2)$. Here, the first and second particle momentum operators are $\hat{p}_1 = -i\hbar\partial/\partial y_1$ and $\hat{p}_2 = -i\hbar\partial/\partial y_2$, where the eigenvalue problems are $\hat{p}_1\varphi_k(y_1) = (\hbar k)\varphi_k(y_1)$ and $\hat{p}_2\varphi_{-k}(y_2) = (-\hbar k)\varphi_{-k}(y_2)$. However, in order to incorporate a spatial separation, $\Delta y = y_1 - y_2$, between the particles, the appropriate two-particle wave function (using the Dirac delta function) is

$$\psi(y_1, y_2) = \delta(y_1 - y_2 - \Delta y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk e^{ik(y_1 - y_2 - \Delta y)}. \tag{1}$$

Ultimately, it is important to note that this construction of the two-particle wavefunction can be written as an infinite spectrum (or Fourier integral representation) of momentum eigenstates of the first particle, $\varphi_k(y_1)$, with momentum eigenvalue $p_1 = \hbar k = p$, where

$$\psi(y_1, y_2) = \int_{-\infty}^{\infty} dk\left[\psi_k(y_2)\right]\varphi_k(y_1), \tag{2}$$

which incorporates the spectral coefficient of the first particle momentum eigenfunction of

$$\psi_k(y_2) = \frac{1}{\sqrt{2\pi}} e^{-ik(y_2 + \Delta y)} = e^{-ik\Delta y}\varphi_{-k}(y_2), \tag{3}$$

which is also a momentum eigenfunction of the second particle, $\varphi_{-k}(y_2)$, with eigenvalue of $p_2 = -\hbar k = -p = -p_1$. Consequently, the two-particle wavefunction, $\psi(y_1, y_2)$, is the desired infinite linear superposition of momentum eigenstates of the first particle with momentum $p_1 = p$, and momentum eigenstates of the second particle, with opposite momentum of $p_2 = -p_1 = -p$.

At this point in the analysis it should be clear that if the momentum of the first particle is measured, resulting in the momentum value $p_1 = p$, then the infinite linear superposition has effectively collapsed after the measurement has occurred to the reduced wave functionresult, where

$$\psi_{after\ p_1}(y_1, y_2) \to \psi_k(y_2)\varphi_k(y_1) = e^{-ik\Delta y}\varphi_{-k}(y_2)\varphi_k(y_1). \tag{4}$$

It should be noted that this resultant wave function is a momentum eigenstate of the first particle, with momentum $p_1 = p$, as well as being a momentum eigenstate of the second particle, with momentum $p_2 = -p_1 = -p$. This result allows for the notion that one can predict the precise momentum of the second particle (without disturbing the second particle) by measuring the momentum of the first particle, which are precisely anti-correlated.

Finally, in order to finish the EPR argument, it is useful to rewrite the two-particle wave function using an alternate position eigenstate basis expansion, where the original two-particle wave function, equation (1), was represented as a momentum eigenfunction expansion of the first particle (over all possible momentum values of $p = \hbar k$), equation (2). Instead, consider the position eigenfunction, $\delta(y_1 - y)$, expansion of the first particle (over all possible position values of y). Note that the position operator of the first particle, $\hat{y}_1$, leads to the eigenvalue problem, where $\hat{y}_1\delta(y_1 - y) = y\delta(y_1 - y)$, with the position eigenvalue of $y_1 = y$. Consequently, the two-particle wave function can equivalently be expressed as

$$\psi(y_1, y_2) = \delta(y_1 - y_2 - \Delta y) = \int_{-\infty}^{\infty} dy\left[\delta(y_2 - y + \Delta y)\right]\delta(y_1 - y). \tag{5}$$

The conclusion is that the spectral coefficient of the first particle position eigenfunction, $\delta(y_1 - y)$, is $\delta(y_2 - y + \Delta y)$, which is also an eigenfunction of the second particle position operator, $\hat{y}_2$, where

$\hat{y}_2\delta(y_2 - y + \Delta y) = (y - \Delta y)\delta(y_2 - y + \Delta y)$, with position eigenvalue of $y_2 = y - \Delta y$. Consequently, once again, this is the desired construct for the two-particle wave function, as it is an infinite linear superposition of position eigenstates of the first particle with position $y_1 = y$, and corresponding position eigenstates of the second particle with a position of $y_2 = y - \Delta y$.

In a similar fashion as the collapse of the wave function upon momentum measurement, if instead the position measurement of the first particle occurs, with a position value of $y_1 = y$, then the infinite linear superposition has effectively collapsed after measurement has occurred to the reduced wave function result, where

$$\psi_{after\ y_1}(y_1, y_2) \to \delta(y_2 - y + \Delta y)\delta(y_1 - y). \tag{6}$$

It should be noted that the resultant wave function is an eigenfunction of position of the first particle, with position valueof $y_1 = y$, as well as being an eigenfunction of position of the second particle, with position value of $y_2 = y - \Delta y$. Once again, this is essentially the notion that one can predict the precise position of the second particle (without disturbing the second particle) by measuring the position of the first particle, which are precisely correlated as shown. Also, as a simplification of this analysis, if the separation is $\Delta y = 2y$, then the second particle position is precisely the opposite of the first particle position, where $y_2 = -y$, and the simplified collapsed wave function is

$$\psi_{after\ y_1}(y_1, y_2) \to \delta(y_2 + y)\delta(y_1 - y). \tag{7}$$

Ultimately, the proposed conclusion of the EPR analysis is that it appears that by considering two different measurements on the first particle (the momentum and the position), then the corresponding values of the second particle (which are precisely anti-correlated with the measurements of the first particle) appear to be precise and simultaneous aspects of reality. However, since the momentum and position values are eigenvalues of two non-commuting operators, where the commutation relation is $[\hat{y}, \hat{p}] = \hat{y}\hat{p} - \hat{p}\hat{y} = i\hbar$, and the quantum mechanical wave function description is not allowed to predict simultaneously such non-commuting results (due to the uncertainty principle which is associated with the non-commutation relation), it was concluded that the wave function description is not complete, since a more complete model should include such knowledge of the two eigenvalues of momentum and position. In retrospect, the error with the EPR logic is that, as with all quantum systems, if a momentum measurement of the second particle was actually used to verify the prediction of the momentum (which was determined using the first particle momentum), then the position realization of the second particle is immediately destroyed, which is then no longer correlated with the first particle position. Of course, a similar but inverseproblem could also be approached by measuring the position of the second particle (which was predicted from the position of the first particle), then the momentum realization of the second particle is immediately destroyed, which is then no longer correlated with the first particle momentum. Consequently, as always in quantum systems, one cannot know simultaneous eigenvalues of non-commuting operators (such as momentum and position).

As a preview of the discussion to be provided in section 6, it is useful to consider that the EPR premise pertaining to their criteria for elements of physical reality is simply incorrect for quantum systems, which have an inherent complexity which goes beyond classical systems. Simply put, Einstein's statement was that: "If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity." However, as is shown above, even though the momentum and the position of the second particle could have been precisely predicted in advance of measurements made on the second particle, by instead measuring the first particle (due to the precise anti-correlation), in quantum systems, as a result of the inherent complexity (which will be explained in section 6), it is not true that these physical quantities are actually simultaneous elements of physical reality until the measurements are made on the quantum system. This problem of assigning elements of physical reality will be described later in analogy to predicting values of emergent reality associated with nonlinear systems which exhibit deterministic chaos (which are in practice actually non-predictable values). Consequently, since the EPR criteria for elements of reality for complex quantum systems is not valid, the EPR argument claiming that the quantum mechanical wave function is not complete was not sound; however, as pointed out in section 7, the intent of the EPR argument was sound, and due to the need to describe more completely the complexity of quantum systems, quantum mechanics in general will be described as ultimately not being complete.

3. Brief Review of the Bohm Spin Proposal

In analogy to the EPR analysis, Bohm [3] put forth in 1951 the concept of analyzing spin measurements on a pair of spin one-half particles which formed a spin zero singlet two-particle state. The two particles were assumed to separate without changing the combined spin zero state, where spin measurements of the separate particles were done along potentially two different orthogonal measurement directions. In a similar fashion to the EPR analysis, the realization of the spin measurements of the separate particles along two non-commuting spin operator directions was considered. For example, consider two spin one-half particles (labelled as 1 and 2) which are separated away from each other along the y axis, where spin measurements of each separate particle can be performed in two orthogonal directions, along the positive z axis and the positive x axis directions, which are represented by the unit vectors $\hat{\mathbf{z}}$ and $\hat{\mathbf{x}}$, as shown in Fig. 2.

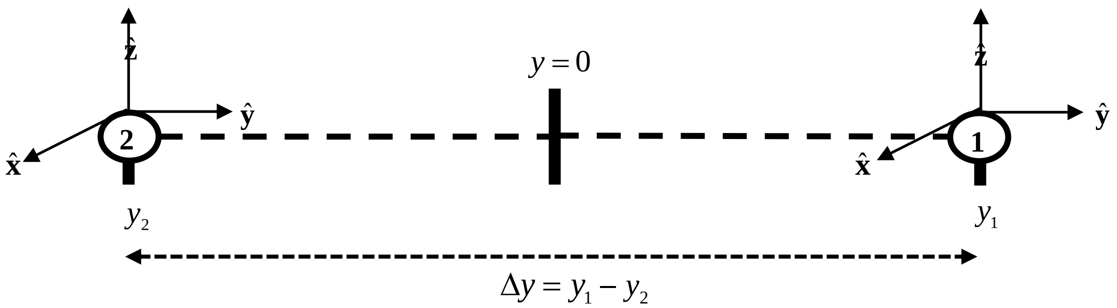

Fig2. *Bohm model of a spin zerosinglet state of two separate particles.*

The general singlet state wave function can be constructed usingspin up $(+)$ and spin down $(-)$ basis functions, $\left|\pm\hat{\mathbf{a}}\right\rangle_1$, along anyunit vector $\hat{\mathbf{a}}$ direction, for the first (labeled as 1) particle, and a similar basis for the second (labeled as 2) particle, where the singlet two-particle state is

$$\left|\psi\right\rangle = \left(\left|+\hat{\mathbf{a}}\right\rangle_1 \left|-\hat{\mathbf{a}}\right\rangle_2 - \left|-\hat{\mathbf{a}}\right\rangle_1 \left|+\hat{\mathbf{a}}\right\rangle_2\right)/\sqrt{2}\,. \tag{8}$$

As a specific example of such a singlet spin state, it is common to use an orthonormal basis along the $\hat{\mathbf{z}}$ direction, with a two component matrix analysis, where the spin up and down states are labelled as $\left|+\hat{\mathbf{z}}\right\rangle = \left|\alpha_z\right\rangle = \alpha_z = \begin{pmatrix}1\\0\end{pmatrix}$ and $\left|-\hat{\mathbf{z}}\right\rangle = \left|\beta_z\right\rangle = \beta_z = \begin{pmatrix}0\\1\end{pmatrix}$, respectively, and where

$$\left\langle+\hat{\mathbf{z}}\right|\left|+\hat{\mathbf{z}}\right\rangle = 1, \quad \left\langle-\hat{\mathbf{z}}\right|\left|-\hat{\mathbf{z}}\right\rangle = 1, \quad \left\langle-\hat{\mathbf{z}}\right|\left|+\hat{\mathbf{z}}\right\rangle = 0\,. \tag{9}$$

Consequently, the entangled two-particle spin state is

$$\left|\psi\right\rangle = \left(\alpha_{z1}\beta_{z2} - \beta_{z1}\alpha_{z2}\right)/\sqrt{2}\,. \tag{10}$$

Using an equivalent complete orthonormal basis along the $\hat{\mathbf{x}}$ direction, where $\left|+\hat{\mathbf{x}}\right\rangle = \left|\alpha_x\right\rangle = \alpha_x = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ and $\left|-\hat{\mathbf{x}}\right\rangle = \left|\beta_x\right\rangle = \beta_x = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$, the entangled two-particle spin state is

$$\left|\psi\right\rangle = \left(\alpha_{x1}\beta_{x2} - \beta_{x1}\alpha_{x2}\right)/\sqrt{2}\,. \tag{11}$$

The general spin vector operator, $\hat{\mathbf{S}} = \frac{\hbar}{2}\hat{\boldsymbol{\sigma}}$, can be normalized using the Pauli spin matrices, which have spin eigenvalues of ± 1, instead of $\pm\hbar/2$, where the x, y, z component matrix forms are

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \hat{\sigma}_y = i\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{12}$$

Note that the eigenvalue problems for the $\hat{\sigma}_z$ operator are $\hat{\sigma}_z\alpha_z = (+1)\alpha_z$ and $\hat{\sigma}_z\beta_z = (-1)\beta_z$, and the eigenvalue problems for the $\hat{\sigma}_x$ operator are $\hat{\sigma}_x\alpha_x = (+1)\alpha_x$ and $\hat{\sigma}_x\beta_x = (-1)\beta_x$, where the +1 and -1 spin eigenvalues should be noted.

In a similar fashion as the EPR analysis, the two-particle singlet spin statewill be employed, using two different bases, equations (10) and (11), which are associated with two non-commuting spin operators, where the commutation relation is $[\hat{\sigma}_z, \hat{\sigma}_x] = \hat{\sigma}_z\hat{\sigma}_x - \hat{\sigma}_x\hat{\sigma}_z = 2i\hat{\sigma}_y$, in order to consider precise and simultaneous orthogonal spin component realities (with one along $\hat{\mathbf{z}}$ and the other along $\hat{\mathbf{x}}$) for the two separated particles. In order to explore this conjecture, it is first useful to demonstrate that the singlet spin zero state leads to spin measurements of the two entangled particles which are precisely anti-correlated. Using the spin operators, equation (12), applied to the $\hat{\mathbf{z}}$ basis spin states (α_z and β_z), the following useful spin operator results can be obtained, where

$$\hat{\sigma}_x\alpha_z = \beta_z, \quad \hat{\sigma}_x\beta_z = \alpha_z, \quad \hat{\sigma}_y\alpha_z = i\beta_z, \quad \hat{\sigma}_y\beta_z = -i\alpha_z. \tag{13}$$

With these tools, the eigenvalue problems for the squared vector and the $\hat{\mathbf{z}}$ component of the combined two-particle spin operator, $\hat{\sigma}^2 = (\hat{\boldsymbol{\sigma}}_1 + \hat{\boldsymbol{\sigma}}_2)^2 = \hat{\sigma}_1{}^2 + \hat{\sigma}_2{}^2 + 2\hat{\boldsymbol{\sigma}}_1 \cdot \hat{\boldsymbol{\sigma}}_2$ and $\hat{\sigma}_z = \hat{\sigma}_{z1} + \hat{\sigma}_{z2}$, for the singlet state, $|\psi\rangle$ in equation (10), provide zero total squared and $\hat{\mathbf{z}}$ component normalized spin eigenvalues ($\sigma^2 = \sigma_z = 0$), where $\hat{\sigma}^2|\psi\rangle = 0$ and $\hat{\sigma}_z|\psi\rangle = 0$. In fact, the two-particle total vector spin squared and the totalspin component along any $\hat{\mathbf{a}}$ direction is zero for the general singlet state wave function, given in equation (8), which is of course also true for the $\hat{\mathbf{x}}$component basis, in equation (11), where $\sigma^2 = \sigma_x = 0$. The utility of this general combined two-particle spin result for the singlet state is that a precise prediction for the spin along any direction of the second particle can be determined in advance by measuring the spin of the first particle along the same direction (without disturbing the second particle), as the spin components are precisely anti-correlated.

Finally, using a similar discussion as was done for the EPR analysis of two entangled particles, associated with the momentum and position measurements of two non-commuting operators ($\hat{p}$ and $\hat{y}$), the singlet state spin zero pair of particles can be analyzed in terms of potentially performing spin measurements along two different orthogonal directions (the $\hat{\mathbf{z}}$ and $\hat{\mathbf{x}}$ directions), associated with two non-commuting spin operators ($\hat{\sigma}_z$ and $\hat{\sigma}_x$). The argument then proceeds by considering a $\hat{\mathbf{z}}$ component spin measurement of the first particle, and if the result is spin up, the initial two-particle state in the $\hat{\mathbf{z}}$ basis, equation (10), effectively collapses to

$$|\psi\rangle_{after\ \alpha_{z1}} \to \alpha_{z1}\beta_{z2}, \tag{14}$$

which is not only a spin up eigenfunction for the first particle along the $\hat{\mathbf{z}}$ direction, where $\hat{\sigma}_{z1}\alpha_{z1} = (+1)\alpha_{z1}$, with eigenvalue $\sigma_{z1} = +1$, it is also a spin down eigenfunction for the second particlealong the $\hat{\mathbf{z}}$ direction, where $\hat{\sigma}_{z2}\beta_{z2} = (-1)\beta_{z2}$, with eigenvalue $\sigma_{z2} = -1$. Of course, if the spin measurement result for the first particle was instead spin down, then the singlet state collapses such that the spin result for the second particle would be spin up, where the anti-correlated eigenvalues would be $\sigma_{z1} = -1$ and $\sigma_{z2} = +1$. However, to finish the argument, it is useful to alternatively consider an $\hat{\mathbf{x}}$ component spin measurement of the first particle, and if the result is spin up, the initial two-particle state in the $\hat{\mathbf{x}}$ basis, equation (11), effectively collapses to

$$|\psi\rangle_{after\ \alpha_{x1}} \to \alpha_{x1}\beta_{x2}, \tag{15}$$

which is not only a spin up eigenfunction for the first particle along the $\hat{\mathbf{x}}$ direction, where $\hat{\sigma}_{x1}\alpha_{x1} = (+1)\alpha_{x1}$, with eigenvalue $\sigma_{x1} = +1$, it is also a spin down eigenfunction for the second particle along the $\hat{\mathbf{x}}$ direction , where $\hat{\sigma}_{x2}\beta_{x2} = (-1)\beta_{x2}$, with eigenvalue $\sigma_{x2} = -1$. Of course, if the spin measurement result for the first particle was instead spin down, then the singlet state collapses such that the spin result for the second particle would be spin up, where the anti-correlated eigenvalues would be $\sigma_{x1} = -1$ and $\sigma_{x2} = +1$.

As in the EPR argument, since the singlet spin zero state of two entangled particles can be equally represented using two different spin bases, associated with two non-commuting spin operators, $\hat{\sigma}_z$ and $\hat{\sigma}_x$, it would appear that a precise prediction of the second particle spin in the $\hat{\mathbf{z}}$ direction could be achieved by measuring the $\hat{\mathbf{z}}$ component spin of the first particle (without disturbing the second particle), or instead a precise prediction of the second particle spin in the $\hat{\mathbf{x}}$ direction could be achieved by measuring the $\hat{\mathbf{x}}$ component spin of the first particle (without disturbing the second particle). However, as in the EPR argument, since a quantum mechanical spin state cannot allow for simultaneous eigenvalues of two non-commuting operators ($\hat{\sigma}_z$ and $\hat{\sigma}_x$), and it appears that the precise spin information of the second particle is objectively real for these two orthogonal spin values (as described above), then the quantum mechanical wave function appears not to be complete, as it should have this extra spin information encoded in its description of reality.

However, just as with the EPR argument, the error with this singlet spin state precise anti-correlation logic is that, as with all quantum systems, if a spin measurement of the second particle was actually used to verify the prediction of the spin along the $\hat{\mathbf{z}}$ direction (which was determined using the first particle spin measurement), then the spin realization along the $\hat{\mathbf{x}}$ direction of the second particle is immediately destroyed, which is then no longer correlated with the first particle spin in the $\hat{\mathbf{x}}$ direction. Of course, a similar but inverse problem could also be approached by measuring the spin of the second particle in the $\hat{\mathbf{x}}$ direction (which was predicted from the spinmeasurement of the first particle), then the spin realization of the second particle in the $\hat{\mathbf{z}}$ direction is immediately destroyed, which is then no longer correlated with the first particle spin in the $\hat{\mathbf{z}}$ direction. Consequently, as always in quantum systems, one cannot know simultaneous eigenvalues of non-commuting operators (such as two orthogonal components of spin).

As a final comment on the conclusions associated with the precise anti-correlation between spin measurements for a singlet state pair of particles, and even though the spin state concept was devised by Bohm to further explore the EPR argument, it was reasonably clear from his analysis [see reference 3, specifically in section 22.18] that he did not agree with the EPR argument of assuming that spin values are objectively real prior to measurement, as he described them as potentialities, to be determined by measurement. In fact, this should be clear from the following quote of Bohm's, "Thus, for a given atom, no component of the spin of a given variable exists with a precisely defined value, until interaction with a suitable system, such as a measuring apparatus, has taken place." Furthermore, Bohm proceeds to claim that quantum theory is inconsistent with the existence of hidden variables, which has been historically first associated with the EPR analysis. To support Bohm's stance on the inconsistency of hidden variables, it is useful to consider the following quote, "Moreover, the present form of quantum theory implies that the world cannot be put into a one-to-one correspondence with any conceivable kind of precisely defined mathematical quantities, and that a complete theory will always require concepts that are more general than that of analysis into precisely defined elements." It should be noted that this conjecture of Bohm's was profound, as it was not clearly shown until the advent of the Bell inequality analysis that a local hidden variable theory of spin cannot be constructed (as shown in section 4). Furthermore, as discussed in section 5, the infinite complexity of spin can be shown to be consistent with an emergent reality that results upon measurement, in a similar fashion as the deterministic chaos exhibited by classical nonlinear dissipative systems. It is also interesting to note that this too seems to be predicted by Bohm, associated with another profound quote of his, "We may probably expect that even the more general types of concepts provided by the present quantum theory will also ultimately be found to provide only a partial reflection of the infinitely complex and subtle structure of the world."

4. Brief Review of the Bell Inequality Analysis

With the objective of exploring the possibility of a local hidden variable theory of spin, Bell [4] put forth in 1964 a generalization of the spin correlation experiments that had originally been suggested by Bohm (as described in section 3).The proposal was for two spin ½ particles, which formed an original singlet pair state, as given in equation (8), to separate to a sufficient distance away from each other (without disturbing the spin structure of each particle), where simultaneous spin measurements are then made on the particles (beyond the light cone, so that there would be no way, within the speed of light limit, for the separate particle measurements to effect each other). Specifically, as shown in Fig. 3, the first particle is measured in the $\hat{\mathbf{a}}$ direction, while the second particle is measured in the $\hat{\mathbf{b}}$ direction, where the angle of separation, θ, is associated with the dot product of the two unit vector directions, as $\hat{\mathbf{a}}\cdot\hat{\mathbf{b}}=\cos(\theta)$.With the use of standard quantum mechanical theory, the expectation value for the product of the two spin measurements can easily be derived (as shown below). Denoting the normalized (to ± 1) spin measurement of the first particle as A, and the normalized spin measurement of the second particle as B, with the use of appropriate vector spin operators for the first particle as $\hat{\boldsymbol{\sigma}}_1$, and for the second particle as $\hat{\boldsymbol{\sigma}}_2$, the quantum mechanical result for the spin product expectation value, $P_{QM}(\theta)$, is

$$P_{QM}(\theta)=\langle AB\rangle=\langle\psi|(\hat{\mathbf{a}}\cdot\hat{\boldsymbol{\sigma}}_1)(\hat{\mathbf{b}}\cdot\hat{\boldsymbol{\sigma}}_2)|\psi\rangle=-\hat{\mathbf{a}}\cdot\hat{\mathbf{b}}=-\cos(\theta). \tag{16}$$

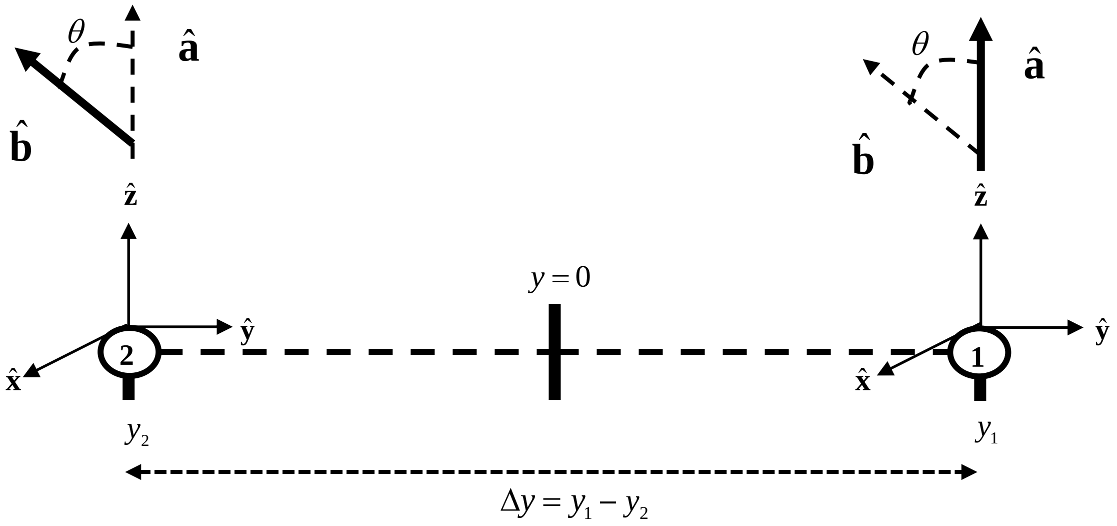

Fig3. *Bell model of the spin product expectation value analysis of singlet state particles.*

The result of the Bell analysis (as shown below) was that a local hidden variable theory for the spin product expectation value cannot be constructed to agree with the quantum mechanical result, equation (16). However, unfortunately the conclusion of the Bell inequality analysis, which was based on a local spin model, led much of the physics community to conclude that a nonlocal process between the separated particles must be responsible for the unusual spin correlation shown in equation (16). Nevertheless, recent analysis of the spin correlation [1], put forth in 2015, has shown that another explanation of the result can be made which utilizes a local mechanism, which would have been much more pleasing to Einstein than the proposed nonlocal approach. Furthermore, in order to achieve the recent alternate understanding of the spin correlation between separated spin ½ particles it was necessary to draw an analogy to the emergent behavior found in classical nonlinear dissipative systems which exhibit deterministic chaos. Here, it should be noted that, not only was Einstein not privy during the 1930s to the computational simulations that are needed for studies of chaotic systems, but also the more recent computational simulation technologythat was needed to understand deterministic chaos, which began during the 1970s and 1980s,was not available to Bell when his spin correlation analysis was first proposed. Consequently, it is not surprising that the physics community had been led down the unfortunate path of exploring a nonlocal construct to explain the spin correlation results since the advent of the Bell analysis.

To provide a brief overview of the spin correlation analysis, it is first useful to review the standard quantum mechanical approach to the problem. For simplicity, although this approach is quite general, it is simplest to start with the entangled two-particle singlet spin state in the $\hat{\mathbf{z}}$ basis, equation (10), with the choice of measurement directions being $\hat{\mathbf{a}}=\hat{\mathbf{z}}$ and $\hat{\mathbf{b}}=\hat{\mathbf{x}}\sin\theta+\hat{\mathbf{z}}\cos\theta$, so that the spin product expectation value can be calculated as

$$P_{QM}\left(\theta\right)=\langle\psi|\left(\hat{\mathbf{a}}\cdot\hat{\boldsymbol{\sigma}}_1\right)\left(\hat{\mathbf{b}}\cdot\hat{\boldsymbol{\sigma}}_2\right)|\psi\rangle=\langle\psi|\left(\hat{\sigma}_{z1}\right)\left(\hat{\sigma}_{x2}\sin\theta+\hat{\sigma}_{z2}\cos\theta\right)|\psi\rangle. \tag{17}$$

Using the singlet state, $|\psi\rangle=\left(\alpha_{z1}\beta_{z2}-\beta_{z1}\alpha_{z2}\right)/\sqrt{2}$, the orthonormal inner products that occur after the $\hat{\sigma}_{z1}$ operator is applied to the particle 1 cross states, using equation (9), and the spin operator identities, equation (13), applied to the particle 2 states, with the needed expectation value results, $\langle\beta_{z2}|\hat{\sigma}_{x2}|\beta_{z2}\rangle=0$, $\langle\alpha_{z2}|\hat{\sigma}_{x2}|\alpha_{z2}\rangle=0$, $\langle\beta_{z2}|\hat{\sigma}_{z2}|\beta_{z2}\rangle=-1$, and $\langle\alpha_{z2}|\hat{\sigma}_{z2}|\alpha_{z2}\rangle=1$, provide the quantum mechanically derived spin product expectation valuein agreement with equation (16), as

$$\begin{aligned}P_{QM}\left(\theta\right)&=\sin\theta\langle\beta_{z2}|\hat{\sigma}_{x2}|\beta_{z2}\rangle/2+\cos\theta\langle\beta_{z2}|\hat{\sigma}_{z2}|\beta_{z2}\rangle/2\\&\quad-\sin\theta\langle\alpha_{z2}|\hat{\sigma}_{x2}|\alpha_{z2}\rangle/2-\cos\theta\langle\alpha_{z2}|\hat{\sigma}_{z2}|\alpha_{z2}\rangle/2=-\cos\theta\end{aligned}. \tag{18}$$

Here, it is important to note that this expectation value result, when applied to spin measurements in the same direction, where $\hat{\mathbf{a}}=\hat{\mathbf{b}}$ so that $\theta=0$, as was shown in section 3 for spin measurements of both separated particles in either the $\hat{\mathbf{z}}$ or the $\hat{\mathbf{x}}$ directions, shows that there is a precise anti-correlation between spin results, where $P_{QM}\left(\theta=0\right)=-1$. In fact, this was the result that was the original impetus for the EPR/Bohm precise anti-correlation analysis between separated particles, which led to the discussion of the possible existence of non-commuting operator associated objective realities, which were assumed to exist prior to the verifying measurements actually taking place. Furthermore, it will be shown below that the most interesting aspect of the spin correlation result, equation (16), is for the case that the spin measurement directions are not the same, that is for a general separation angle, $\theta\neq0$, between measurement directions. In this case, the Bell inequality analysis will demonstrate that a local hidden variable theory for the spin product expectation value cannot be constructed to agree with quantum mechanics, given by equation (16).

The next step in the demonstration that the quantum mechanically derived correlation between separated spin measurements of singlet state particles is quite unusual, given by equation (16), is achieved by considering the precise local hidden variable model of spin proposed in the Bell analysis [4]. Specifically, for a hidden variable parameter value of λ, it is assumed that the hidden variable spin of the first and second particles were precisely given by $A\left(\hat{\mathbf{a}},\lambda\right)$ and $B\left(\hat{\mathbf{b}},\lambda\right)$, when the measurement directions are $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$, respectively. To be consistent with quantum mechanics, these functions can only take on plus or minus one values, as

$$A\left(\hat{\mathbf{a}},\lambda\right)=\pm1, B\left(\hat{\mathbf{b}},\lambda\right)=\pm1. \tag{19}$$

In order to compare the quantum mechanically derived expectation value of the product of the spin measurements of the two particles with a hidden variable prediction, a general non-negative probability density, $\rho\left(\lambda\right)$, with $\rho\left(\lambda\right)\geq0$, for occurrence of the hidden variable, λ, was proposed, which has the usual integrated constraint of being unity, where

$$\int d\lambda\rho\left(\lambda\right)=1. \tag{20}$$

Consequently, the hidden variable proposal for the spin product expectation value was given by

$$P\left(\hat{\mathbf{a}},\hat{\mathbf{b}}\right)=\int d\lambda\rho\left(\lambda\right)A\left(\hat{\mathbf{a}},\lambda\right)B\left(\hat{\mathbf{b}},\lambda\right). \tag{21}$$

However, since there is a precise anti-correlation between spin values for measurement directions being the same, $\hat{\mathbf{a}}=\hat{\mathbf{b}}$, where

$$B\left(\hat{\mathbf{b}},\lambda\right)=-A\left(\hat{\mathbf{b}},\lambda\right), \tag{22}$$

thehidden variable theory spin product expectation value, equation (21), can also be written as

$$P\left(\hat{\mathbf{a}},\hat{\mathbf{b}}\right)=-\int d\lambda\rho(\lambda)A(\hat{\mathbf{a}},\lambda)A\left(\hat{\mathbf{b}},\lambda\right). \qquad (23)$$

The Bell inequality analysis proceeded by considering three general unit vector measurement directions, $\hat{\mathbf{a}},\hat{\mathbf{b}},\hat{\mathbf{c}}$, and with the use of equation (23), a useful equation results, where

$$P\left(\hat{\mathbf{a}},\hat{\mathbf{b}}\right)-P(\hat{\mathbf{a}},\hat{\mathbf{c}})=-\int d\lambda\rho(\lambda)\left[A(\hat{\mathbf{a}},\lambda)A\left(\hat{\mathbf{b}},\lambda\right)-A(\hat{\mathbf{a}},\lambda)A(\hat{\mathbf{c}},\lambda)\right]. \qquad (24)$$

In addition, since $\left[A\left(\hat{\mathbf{b}},\lambda\right)\right]^2=1$, this equation can also be written as

$$P\left(\hat{\mathbf{a}},\hat{\mathbf{b}}\right)-P(\hat{\mathbf{a}},\hat{\mathbf{c}})=-\int d\lambda\rho(\lambda)\left[1-A\left(\hat{\mathbf{b}},\lambda\right)A(\hat{\mathbf{c}},\lambda)\right]\left[A(\hat{\mathbf{a}},\lambda)A\left(\hat{\mathbf{b}},\lambda\right)\right]. \qquad (25)$$

By taking the absolute value of both sides of this expression, and using the maximum value of the last term on the right hand side, where $\left[A(\hat{\mathbf{a}},\lambda)A\left(\hat{\mathbf{b}},\lambda\right)\right]_{\max}=1$, the needed inequality results,

$$\left|P\left(\hat{\mathbf{a}},\hat{\mathbf{b}}\right)-P(\hat{\mathbf{a}},\hat{\mathbf{c}})\right|\le\int d\lambda\rho(\lambda)\left[1-A\left(\hat{\mathbf{b}},\lambda\right)A(\hat{\mathbf{c}},\lambda)\right]. \qquad (26)$$

Ultimately, using equations (20) and (23), the well-known Bell inequality results, where

$$1+P\left(\hat{\mathbf{b}},\hat{\mathbf{c}}\right)\ge\left|P\left(\hat{\mathbf{a}},\hat{\mathbf{b}}\right)-P(\hat{\mathbf{a}},\hat{\mathbf{c}})\right|. \qquad (27)$$

The key result of the Bell analysis is that the inequality, equation (27), is violated if the quantum mechanically derived spin product expectation values, equation (16), are used in the expression, in place of the hidden variable proposed spin product expectation values, equation (21). To verify this result it is simplest to show an example of the violation. For example, if the three unit vector measurement directions, $\hat{\mathbf{a}},\hat{\mathbf{b}},\hat{\mathbf{c}}$, are in a plane and are each successively separated by sixty degrees, and the quantum mechanical results, equation (16), are used in equation (27), where

$$P\left(\hat{\mathbf{a}},\hat{\mathbf{b}}\right)\Rightarrow-\cos\left(60^0\right)=-0.5,\ P(\hat{\mathbf{a}},\hat{\mathbf{c}})\Rightarrow-\cos\left(120^0\right)=0.5,\ P\left(\hat{\mathbf{b}},\hat{\mathbf{c}}\right)\Rightarrow-\cos\left(60^0\right)=-0.5, \qquad (28)$$

then the Bell inequality, equation (27), fails since the inequality is not correct, as shown here,

$$1+P\left(\hat{\mathbf{b}},\hat{\mathbf{c}}\right)\ge\left|P\left(\hat{\mathbf{a}},\hat{\mathbf{b}}\right)-P(\hat{\mathbf{a}},\hat{\mathbf{c}})\right|\Rightarrow0.5\ngeq1.0. \qquad (29)$$

The profound conclusion of this analysis is that the local hidden variable model of the spin product expectation value, equation (21), cannot be constructed to agree with quantum mechanics, equation (16), where

$$P\left(\hat{\mathbf{a}},\hat{\mathbf{b}}\right)\neq-\hat{\mathbf{a}}\cdot\hat{\mathbf{b}}=-\cos(\theta)=P_{QM}(\theta). \qquad (30)$$

It is important to note that the bizarre conclusion of the original Bell inequality publication[4] was that, in order for such a local hidden variable spin construct, equation (21), to agree with quantum mechanics, equation (16), it was assumed that there must be a mechanism whereby the setting of one spin measuring device can influence the reading on the other device, such that the signal involved must propagate instantaneously (which is a nonlocal construct proposition). However, as pointed out in a recent publicationwhich describes an alternate and local understanding of the correlation between singlet state particles [1], it should be emphasized that this conclusion, equation (30), does not indicate that quantum mechanical spin must be inherently nonlocal, which would have pleased Einstein, as he would have never wanted to give up locality, simply to force a hidden variable theory to agree with quantum mechanical reality. Furthermore, in contrast to the unusual well-known conclusion of the Bell analysis, the conclusion should have simply been stated that the proposed hidden variable model of the spin product expectation value, equation (21), cannot agree with quantum mechanics unless the model includes a nonlocal construct. Consequently, as the use of a nonlocal spin construct is unacceptable (to Einstein as well as to most sound scientists), it should be clear that a simplistic hidden variable theory of spin, which prescribes precise spin values in advance of measurement, as proposed in equation (19), does not represent a valid candidate spin model of reality. Here, it should be noted that this problem of proposing a precise spin model is in direct analogy to the original failed assumption in the EPR publication which indicated that Einstein had assumed that there should be elements of physical reality (the momentum and position measured values, which in this case are analogous to the measured values of spin), which correspond to physical quantities (the predicted momentum and position values, which in this case are analogous tothe precise spin values proposed by the hidden variable model). Nevertheless, even though Einstein's argument was not completely sound, as it was based on the assumption of precise elements of physical reality, Einstein's proposal that quantum mechanics is not complete will be vindicated in sections 6 and 7, after a brief review of the successful (as it agrees with quantum mechanics),alternate, andlocal understanding of the spin correlation between singlet state particles is presented in section 5.

5. Alternative Understanding of Spin Correlation

It is well known that quantum mechanical experiments (using spin ½ particles as well as photons) clearly support the quantum mechanically derived correlation between singlet state pair particles, given by equation (16), which was most definitively verified by the Aspect experiments [8], performed in 1982, that were inconsistent with the Bell inequality, but supportive of quantum mechanics. However, it is unfortunate that the most common explanation of the unusual spin correlation between singlet state separated particlesis to assume that there is a nonlocal interaction between the particles [9-11]. Nevertheless, recently,an alternate explanation of the spin correlation was published [1] which is appealing since it is based on a local theory. Here, it should be noted that it is not surprising that this alternate understanding of the spin correlation was not discovered until recently, as the analysis is based on knowledge that is associated with computational simulations which demonstrate deterministic chaos found in classical nonlinear dissipative systems [12-15]. Clearly, not only would Einstein have no knowledge of the unpredictable aspects of deterministic chaos in the 1930s, but Bell also would have been in the dark on such concepts in the 1960s, as the detailed numerical simulations of chaos did not begin in earnest until at least the 1970s and 1980s.

Given the advantage and experience that a wide variety of scientists and engineers have gained through the exploration of deterministic chaos models, numerous examples of the unpredictable aspects of nature, which are quite pervasive, have been recently explored through detailed computational simulations. Such studies include demonstrations of extreme sensitivity to initial conditions found in nonlinear systems that are as simple as a driven damped pendulum, or as complex as computational fluid dynamics simulations applied to weather prediction, or kinetic theory simulations of plasma turbulence. However, as quantum systems are typically analyzed using the linear Schrödinger equation and linear operator theory, it is not common to apply deterministic chaos concepts to the quantum realm of nature. Nevertheless, as discussed in section 6, this is precisely the new area of research which may lead to far superior understanding and interpretation of quantum phenomena. Thus, with this deterministic chaos knowledge base in mind, the following brief review of the recently proposed alternate explanation of the spin correlation between separated particles begins by considering the possibility that there could be an underlying infinitely complex aspect of spin which emerges upon measurement. Note that this approach of dealing with the measurement problem in quantum mechanics as an emergent behavior, which bypasses the need for a mystical wave function collapse, allows for a natural evolution of the quantum system towards the well-known (but unpredictable) eigenfunctions of linear theory (which allows for a direct connection to standard quantum theory results). To begin the description of the alternate analysis of spin, and as clearly shown in section 4, it is important to note that it is not possible to construct a precise local hidden variable theory of spin for each particle, such as given by equation (19), which agrees with quantum mechanics. However, this does not restrict a proposal for a local statistical model of the spin correlation between the two separated particles, which is designed to agree with quantum mechanics, where the spin product expectation value is given by equation (16), as shown below.

In order to construct a specific model of the spin correlation between two separated particles it is useful to start with the knowledge of the well-known spin product expectation value dictated by quantum mechanics, being $P_{QM}(\theta) = -\cos(\theta)$, as shown in equation (16). In addition, from this result, it is clear that the two spin measurements are: 1) highly anti-correlated, $P_{QM} \to -1$, for small separation angles, $\theta \to 0$; 2) highly correlated, $P_{QM} \to 1$, for large separation angles, $\theta \to \pi$; and 3) completely uncorrelated, $P_{QM} \to 0$, for orthogonal separation angles, $\theta \to \pi/2$.With these statistical concepts of the spin correlation structure for singlet state particles in mind, an alternate type of hidden variable theory for the spin correlation model is proposed; however, in contrast to the usual hidden variable theory that attempts to specify a precise spin structure for each separated particle, the following model simply represents the statistical concepts associated with the spin product of the two separated particles. In addition, since the following spin correlation model for singlet state pair particles incorporates the notion that there could be an underlying infinitely complex aspect of spin, which emerges upon measurement, the phrase Infinite Complexity Hidden Variable (ICHV) theory is coined to represent this concept. Although a similar notation will be used in the following, as was used for the review of the standard hidden variable theory, given in section 4, where the hidden variable parameter was introduced to represent a specific realization of spin, it is important to emphasize that the hidden variable parameter that is used here, λ, is not intended to represent a specific spin of each particle. Furthermore, to help with understanding, and in order to make contact with the notion of the unpredictable and underlying infinitely complex emergent spin structure, one can think of the hidden variable parameter (used here) as representing an infinite variety of initial conditions, which lead to specific spin products of the two particles, AB_{ICHV}, which occur upon measurement. The statistical aspect of the spin model will then be entirely embedded in the probability density function, ρ_{ICHV}, which is associated with a specific hidden variable parameter value, λ, as well as with the angle of separation, θ, between measurements.

The specific spin model proposal includes a normalized spin product, given by $AB_{ICHV}(\lambda)$, where λ is a continuous hidden variable parameter value over the entire infinite domain,

$$-\infty < \lambda < \infty, \tag{31}$$

which have the needed ± 1 normalized spin product values, where

$$AB_{ICHV}(\lambda) = \pm 1. \tag{32}$$

In addition to this spin product function, the spin model also includes an associated non-negative normalized probability density, $\rho_{ICHV}(\theta,\lambda) \geq 0$, for all separation angles of measurement, $0 \leq \theta \leq \pi$, which is properly normalized as a unity infinite integral over the hidden variable, or

$$\int_{-\infty}^{\infty} d\lambda \rho_{ICHV}(\theta,\lambda) = 1. \tag{33}$$

The proposed ICHV spin product expectation value, $P_{ICHV}(\theta)$, is then given by

$$P_{ICHV}(\theta) = \int_{-\infty}^{\infty} d\lambda \rho_{ICHV}(\theta,\lambda) AB_{ICHV}(\lambda). \tag{34}$$

Here, it should be noted that this expectation value construct appears similar to the original hidden variable theory approach, of equation (21); however, the significant difference here is that the probability density, $\rho_{ICHV}(\theta,\lambda)$, in equation (34), must be a function of the separation angle, θ, as well as the hidden variable, λ, and the spin product, $AB_{ICHV}(\lambda)$, is only a function of the hidden variable, λ, as it will take on only plus or minus one values, depending on the hidden variable realization parameter. Furthermore, it should be emphasized that unlike the standard hidden variable theories, which utilize precise spin predictions for each hidden variable parameter value, the hidden variable parameter used here cannot provide precise spin predictions, as it is only connected statistically to the spin product through the probability density function.

Finally, to complete the description of the specific statistical ICHV spin model proposal, let the spin product function be

$$AB_{ICHV}(\lambda) = \begin{cases} +1, & 0 < \lambda < +\infty \\ -1, & -\infty < \lambda < 0 \end{cases},$$

(35)

where the probability density function is

$$\rho_{ICHV}\left(\theta,\lambda\right)=\begin{cases}\dfrac{2}{\sqrt{\pi}}\exp\left\{-\left[\dfrac{\lambda}{\sin^2\left(\theta/2\right)}\right]^2\right\}, & 0<\lambda<+\infty\\ \dfrac{2}{\sqrt{\pi}}\exp\left\{-\left[\dfrac{\lambda}{\cos^2\left(\theta/2\right)}\right]^2\right\}, & -\infty<\lambda<0\end{cases}. \qquad (36)$$

Here, it should be noted that the choice of the probability density function, equation (36), for analytic simplicity, incorporates a Gaussian function structure, which can easily be integrated over the hidden variable. However, it should be clear that other functional forms for the probability density function could be used if desired.

In order to show that these choices for the spin product in combination with the probability density functions are consistent with the quantum mechanical predictions, it is necessary to check the probability density normalization as well as the spin product expectation value result. First, it is clear that the probability density is properly normalized to one, where

$$\begin{aligned}\int_{-\infty}^{\infty} d\lambda\rho_{ICHV}\left(\theta,\lambda\right)&=\int_{0}^{\infty} d\lambda\frac{2}{\sqrt{\pi}}\exp\left\{-\left[\frac{\lambda}{\sin^2\left(\theta/2\right)}\right]^2\right\}+\int_{-\infty}^{0} d\lambda\frac{2}{\sqrt{\pi}}\exp\left\{-\left[\frac{\lambda}{\cos^2\left(\theta/2\right)}\right]^2\right\}\\ &=\sin^2\left(\theta/2\right)+\cos^2\left(\theta/2\right)=1\end{aligned}. \qquad (37)$$

Second, it is also clear that the spin product expectation value, using this ICHV model, equation (34), agrees with quantum mechanics, as given in equation (16), where

$$\begin{aligned}P_{ICHV}\left(\theta\right)&=\int_{-\infty}^{\infty} d\lambda\rho_{ICHV}\left(\theta,\lambda\right)AB_{ICHV}\left(\lambda\right)=\left(+1\right)\int_{0}^{\infty} d\lambda\rho_{ICHV}\left(\theta,\lambda\right)+\left(-1\right)\int_{-\infty}^{0} d\lambda\rho_{ICHV}\left(\theta,\lambda\right)\\ &=\sin^2\left(\theta/2\right)-\cos^2\left(\theta/2\right)=1-2\cos^2\left(\theta/2\right)=-\cos\left(\theta\right)=P_{QM}\left(\theta\right)\end{aligned}. \qquad (38)$$

As an overviewof the key results associated with this ICHV statistical spin model,first and foremost it should be clear that a nonlocal structure is not necessary to achieve the quantum mechanical spin results (although, until now, this has been the predominant assumption). Instead, the quantum mechanical result was achieved by using a strategically chosen statistical model of the spin structure, which incorporates a probability density, equation (36), which is a function of the separation angle between measurements, θ. Here, it should be clear that the spin model is fundamentally local, as the spin structure is built into the model at the outset prior to measurement of either particle. Ultimately, in combination with the ± 1normalized spin product values, equation (35), and the associated probability density, equation (36), the ICHV local spin model leads to the spin product expectation value, equation (38), which reproduces precisely the quantum mechanical result, equation (16),where $P_{ICHV}\left(\theta\right)=-\cos\left(\theta\right)=P_{QM}\left(\theta\right)$. In addition, it is important to note that the ICHV model is offered as an approach to understand a possible explanation for the unusual quantum mechanical results. Specifically, the hidden variable, λ, is utilized in order to incorporate the needed statistics for the spin model so that it matches the quantum mechanical spin product expectation value result without employing exotic concepts, such as the need for a nonlocal spin mechanism. Furthermore, recall that the spin measurement concept employed here is that the spin system in combination with the measuring device is sufficiently complex such that the spin states cannot be predicted precisely in advance. The spin result should be thought of as emergingduring the process of measurement (for example, as found in systems which exhibit deterministic chaos), while statistical spin results can be consistently predicted for the pair of singlet state particles as a function of the separation angle of measurement, θ. Here, it should be noted that it is the hidden variable parameter in the ICHV spin correlation model that mathematically incorporates the unpredictability of a specific spin measurement realization result, possibly representing the infinite variety of initial conditions in the measurement process. Ultimately, in addition to presenting a local spin theory that agrees with quantum mechanics, one of the primary results of the ICHV analysis is to demonstrate that it may be possible in the future to create more sophisticated (nonlinear dynamics) quantum spin models which incorporate the needed unpredictability, but which will also predict the well-known statistical quantum spin results.

6. Discussion of EPR Proposal Using Enhanced Quantum Models

Given the alternate understanding of the spin product expectation value, provided in section 5, it is useful to revisit the EPR proposal. This is done by considering that the quantum measurement process may be better elucidated using emergent behavior concepts, which are traditionally associated with classical systems that exhibit deterministic chaos, but until now, have not been applied to quantum systems. First of all, it is important to recall that the key problem with the EPR argument (as described in section 2), as well as with the analogous Bohm spin measurement analysis (as described in section 3), is that they both assume measurements associated with non-commuting operators are actually simultaneous elements of reality, existing prior to measurements taking place. For the case of the EPR argument, the momentum and the position measurements of two separated entangled particles were consideredusing the concept of a two-particle quantum state, built on a superposition of anti-correlated momentum eigenfunctions. For the case of the Bohm spin analysis, two orthogonal spin component measurements of two separated entangled particles were considered using the concept of a two-particle singlet spin zero quantum state, built on a superposition of anti-correlated spin eigenfunctions.However, for either of these entangled quantum state analyses, as the measurements of the second paired particle could always be precisely predicted prior to measurements, by instead using measurements on the first particle in the pair, it was reasonable to assume that measured values were associated with precise elements of reality, which existed prior to measurements actually occurring. Note that this was the rationale behind the incorrect proposal that Einstein put forth in the EPR argument.

Secondly, it was not until the advent of the Bell inequality analysis, which clearly exhibited the unusual spin correlation between separated singlet state particles, as described in section 4, that spin component values were finally demonstrated as not beingelements of reality, which exist prior to measurements actually occurring. Here, it is important to note that this is precisely why the local and precise hidden variable spin model proposal, equation (19), failed to reproduce the quantum mechanically derived spin product expectation value, equation (16), using the hidden variable construct for the spin product expectation value, equation (21). In addition, it should be noted that the added complexity proposed in the Bell analysis was necessary, where spin values of the separate particles were considered in general measurement directions, which allowed for the (all important) general statistical correlation between spin measurements to be explored, in contrast to the much simpler and precise anti-correlation that occurs for spin measurementsin the same direction. Ultimately, with the knowledge that a precise hidden variable theory of spin cannot be constructed to agree with quantum mechanics, it is important to note that a local spin model can be used to reproduce the quantum mechanical spin product expectation value result, if the model is statistical (as was shown in section 5), where the statistical spin model given by equations (35) and (36), was used to derive the ICHV spin product expectation value result, equation (34). The key point here is that the quantum mechanical results finally make sense if measurements are viewed as an emergent reality, but not viewed as a given reality that exists prior to measurement, which had been falsely assumed in the EPR argument.

Finally,it is important to point out that it was much simpler to analyze two-state spin systems, as was done in the Bell inequality analysis, to show that precise hidden variable theories cannot represent quantum reality, than it would have been to analyze infinite-state traveling wave systems, as was proposed in the EPR analysis. However, as advertised at the outset of this publication, in section 1, the focus in this section will be to connect the emergent behavior concepts, learned from the exploration of the measurement of two-state spin systems, applied to the continuous quantum mechanical wave function systems of traveling particles, as was the focus of the EPR analysis. In either case, the two-state system or thecontinuous wave function system, the key problem with the interpretation of an underlying reality, prior to measurement, is that the measurement process in quantum mechanics is typically described as causing a wave function collapse. In this description of reality, the measured values are associated with the eigenfunctions of the linear operator theory, which is used to model the measuring device, either for spin component state measurements or for momentum state measurements. Here, it should be emphasized that the physics associated with the mystical concept of a wave function collapse is precisely what is needed to understand the emergent behavior of a quantum system that occurs upon measurement, which is the focus of this publication.

In the following, in order to propose how one might apply the emergent behavior knowledge,that was gained by exploring the two-state quantum spin system (addressed in section 5), to the infinite-state, traveling particle, quantum wave function system, it is useful to first remind the reader of the discrete (quantum) aspect of quantum measurements. Here, it is important to recall that the discreteness found in quantum systems is typically expressed as being proportional to the (normalized) Planck constant, $\hbar$. For example, in the case of the two-state spin ½ particle quantum system, the difference of the spin values, between the spin up and down states, is exactly the Planck constant, where $\Delta S_z = \left(+\hbar/2\right) - \left(-\hbar/2\right) = \hbar$. However, even for classically continuous systems, such as the orbital angular momentum, L_z, in the $\hat{\mathbf{z}}$ direction, of an electron orbiting a proton, as found in a hydrogen atom, the discrete size of the possible minimum change in the associated quantum mechanical angular momentum, is also the Planck constant; for example, considering an $m=2$ down to an $m=1$ transition, then $\Delta L_z = L_{m=2} - L_{m=1} = 2\hbar - 1\hbar = \hbar$. Furthermore, this concept of the discreteness size being proportional to the Planck constant is also true in general for any bound quantum mechanical system, whether one is considering changes in linear momentum Δp, or changes in the canonically conjugate spatial coordinate, Δy. Although, for truly infinite size systems, such as an infinite size box along the y coordinate of interest, the discreteness size approaches zero, where the associated quantum mechanical states approach a continuum of states; for example, the momentum states (described in section 2), $\varphi_k\left(y\right)$, incorporate a continuous wavenumber index, k, where the momentum values are also continuous, $p = \hbar k$. Nevertheless, in reality, for any measuring system that one can actually construct, it should be assumed as havinga finite size. Consequently, in the following discussion which addresses an extended analysis of the EPR proposal, we will assume that all measurements of quantum systems will incorporate the usual concept of quantum discreteness, whether it is for measured values of spin, associated with a Stern-Gerlachgradient magnetic field induced spin up or spin down trajectory, as in the Bell proposed spin measurements, or whether it is for measured values of momentum or position, associated with particle trajectories, as in the EPR proposed measurements.

Ultimately, considering the above discussion on thepractical proposal for measurements of quantum discreteness in various systems, here it is assumed that the constraints associated with spin measurements, as well as with particle momentum and position measurements, are essentially the same. Consequently, the notion that was presented in section 5, where it was assumed that the emergent behavior in the spin measurement process is more accurately associated with an underlying highly sophisticated dynamical complexity, in which either the discrete spin up or the spin down states evolve as they are observed (in the Bell spin analysis), will also be applied to the seemingly continuous aspect of the momentum and position measurement of a particle trajectory (in the EPR particle trajectory analysis). In order to reiterate and extend the key conclusions associated with the emergent behavior that occurs during spin measurements (first discussed towards the end of section 5), it is important to recall that it is reasonable to assume that the quantum spin system, in combination with a classical spin measuring device, exhibits a significant overall complexity (as found in the Bell spin analysis). Specifically, it is assumed that the combined system should be modeled using nonlinear dissipative dynamics, which in classical systems, typically exhibit deterministic chaos solutions, which show an extreme sensitivity to initial conditions. Furthermore, employing the ICHV statistical spin model, it is important to recall that the hidden variable parameter, λ, could be thought of as representing the infinite variety of initial conditions which lead to the unpredictable results of the emerging behavior analysis. However, since the linear operator theory for the spin system only has two possible discrete state solutions, the result of the highly complex nonlinear dynamics evolution must be quite simple, with either a spin up or a spin down result.

For the case of the EPR particle trajectory analysis, a similar approach could be used, as was proposed for the Bell spin analysis, by combining the quantum wavefunction analysis of the particle trajectory, with a classical momentum and position measuring device, resulting in a highly complex overall system, used to achieve the observed unpredictable emergent measurement behavior. However, it would be more difficult to devise an overall nonlinear dissipative dynamics model to effectively combine measurements of momentum and position of the particle, within the constraint of the uncertainty principle, than it would be to devise an overall complex spin model that combines two orthogonal components of spin; however, in principle this can be achieved. Furthermore, it is also clearly more difficult to include the numerous possible quantum momentum state and position state results in the EPR trajectory analysis, associated with all the quantized linear operator theory momentum and position state results, than it would be for the two-state spin system; however, once again, in principle this can be achieved. Finally, given these concepts for modeling the unpredictable emergent behavior of the combined momentum and position measurements of two correlated particle trajectories, an extension of the EPR anti-correlation particle trajectory analysis can be achieved to explore the general type of correlations that were addressed in the Bell spin analysis. Specifically, an extension of the EPR analysis could be done to achieve a general combination of momentum and position measurements, in a similar fashion as the Bell analysis extended the simple anti-correlation spin measurements, proposed by Bohm, to include a general combination of two orthogonal spin components for each particle. Given these proposed emergent behavior measurement simulations, it would finally be clear that the proposed existence of canonically conjugate momentum and position values, as being actual elements of physical reality (as Einstein had proposed), prior to measurements occurring, would be shown as an incorrect assumption, just as was shown in the Bell analysis using a hidden variable spin assumption.

7. Conclusion

As noted at the end of section 6, it should now be clear from an emergent measurement behavior perspective, that the momentum and position values are not to be considered as elements of physical reality, prior to measurements actually taking place, associated with the EPR particle trajectorytype of experiments. However, this would have invalidated Einstein’s proposal, as he had assumed that the momentum and position were elements of physical reality, in order that he was able to assert that quantum mechanics should not be considered as a complete description of physical reality. Nevertheless, the conclusion of this publication is that the thrust of his proposal was in fact correct, as it will be shown in the following.

First, it is useful to review some of the relevant conclusions provided throughout this document.

1) As pointed out in section 2, since the EPR proposal dealt with precisely anti-correlated (momentum and position) entangled particles, which are strategically separated in a non-disturbing fashion, it was not clear that, in the limited anti-correlation context of the proposal, the momentum and position were not actually elements of physical reality, as Einstein had proposed. Here, it should be recalled that it is quantum theory which dictates that measurements associated with non-commuting operators cannot be known simultaneously; although, the quantum physics community and experiments still support this quantum theory notion pertaining to nature, just as Bohr had originally used in his arguments against the EPR proposal.

2) As pointed out in section 3, since the Bohm spin proposal also dealt with precisely anti-correlated (spin) entangled particles, which are strategically separated in a non-disturbing fashion, it was also not clear that, in the limited anti-correlation context of the proposal, the two orthogonal components of spin were not actually elements of physical reality. Again, it is quantum theory which postulates that two orthogonal components of spin cannot be known simultaneously; although, once again, this is in agreement with experiments. However, it should be noted that Bohm did not believe that the spin components were elements of physical reality, as he described them as being potentialities until measurements actually take place, which would have been in disagreement with Einstein’s proposal for a hidden variable aspect of the quantum mechanical reality. Nevertheless, it is somewhat ironic that Bohm did eventually believe in a hidden variable concept of quantum mechanics, which he developed extensively starting in the early 1950s; however, his theory was fundamentally nonlocal, which would have been very distasteful to Einstein.

3) As pointed out in section 4, the Bell analysis extended the Bohm spin proposal of separated spin zero singlet state entangled particles, to include general spin measurement directions. Consequently, due to the Bell inequality experiments, as well as the local hidden variable theory of spin used for its development, it was finally and definitively concluded that local hidden variable spin models cannot be designed to agree with quantum reality. However, since that time much of the physics community, including Bell, has assumed that the unusual spin correlation exhibited in the Bell spin analysis is due to an underlying nonlocal construct, which would have also been very disturbing to Einstein. Nevertheless, as a result of the Bell analysis, it is most widely believed that spin components are not actually elements of physical reality until measurements are done. In addition, it is also widely

believed that this argument applies to the EPR proposal, where momentum and position values should not be assumed as elements of physical reality until measurements occur. Consequently, whether for spin or for particle momentum and position trajectory measurements, it was finally quite clear that in the quantum domain of nature, one cannot consider the existence of elements of physical reality as being precise until measurements are actually done.

4) As pointed out in section 5, due to the recent development of the ICHV statistical spin model, the Bell spin experiments can instead be understood more clearly using a local spin concept. Furthermore, it was concluded that the unusual spin correlation between separated particles, which includes precise anti-correlation spin aspects as well as random spin aspects, is better understood as an emergent behavior which results upon measurement. In addition, the statistical aspect of the result incorporates a hidden variable parameter, which could be viewed as representing the infinite variety of initial conditions, which allow for the unpredictable aspects of the correlation between spin measurements. It was further emphasized that the exotic spin results could be modeled in a similar fashion as one models classical nonlinear dissipative systems which exhibit deterministic chaos. In any case, it was once again concluded that the spin values should not be viewed as elements of physical reality until measurements are actually conducted.

5) As pointed out in section 6, the concept of an emergent behavior underlying quantum measurements, as first applied to the Bell spin analysis (given in section 5), was finally applied to the EPR particle trajectory proposal for the measurements of momentum and position. Although the two-state spin approach, which was applied to the Bell analysis of general spin measurements, is quite simple, it was argued that a similar analysis could be applied to the infinite-state aspects of particle trajectories, used to extend the EPR analysis in order to demonstrate the more exotic statistical correlations which would be present for general measurements. It was pointed out that this type of extended analysis of the EPR proposal could be achieved by considering a far more complex approach than is done using linear operator theory for quantum systems. Specifically, it is envisioned that a quantum wave function analysis of the particle trajectory should be combined with a classical momentum and position measurement model, in order to achieve a highly complex overall system model, which would behave in a similar fashion as nonlinear dissipative systems, which exhibit deterministic chaos and the associated extreme sensitivity to initial conditions. In this sense, the non-predictable aspects of the emergent measurement behavior of the quantum system could be correctly modeled. Once again, this could be used to demonstrate that the values of momentum and position of an EPR particle trajectory should not be viewed as elements of physical reality, until the measurements are done. However, most importantly, development and verification of such an extended and complex measurement model, could ultimately be used to provide better understanding of the emergent nature of quantum measurements.

Finally, with all these lessons learned in mind (stated in the five review points given above), it is important to reconsider the intent behind Einstein's original EPR argument, which postulated that quantum mechanics should not be considered as a complete description of physical reality. Clearly, from our knowledge of the sophisticated quantum spin correlations associated with the Bell analysis, it is clear that local hidden variable theories of spin cannot model the quantum spin measurement process. In addition, applying these concepts to an extended EPR proposal of general momentum and position correlations, it would also be reasonable to assume that a local hidden variable theory of the particle trajectories cannot model the quantum particle trajectory measurement process. In either case, elements of a quantum system cannot be considered as physically real, prior to measurements actually occurring, which would have invalidated Einstein's EPR proposal. However, it is important to keep in mind that at the time of the EPR proposal, Einstein was limited to only considering a quantum linear operator theory proposal for the measurement process. Which, as usual in traditional quantum mechanics, must be supplemented by a mystical wave function collapse to the linear operator states, when measurements occur. Furthermore, Einstein was not privy at that time to our recent knowledge base associated with deterministic chaos, which is most readily exhibited using numerical simulations of nonlinear dissipative systems. Consequently, at that time, it would not have been a logical proposal of Einstein's to consider an extension of the quantum analysis, in order to include a measurement process which could be understood as an unpredictable emergent behavior, modeled using a highly complex nonlinear dynamic system. Nevertheless, it seems clear that Einstein was correct in proposing that quantum mechanics, as modeled using linear operator theory, is not complete. Ultimately, it is reasonable to assume that Einstein was envisioning the need for a far more complex model of the quantum realm of reality, even if his original EPR proposal was not sufficient to explain this profound conjecture.

ACKNOWLEDGEMENTS

I am thankful for the generous support from the Dept. of Physics and Astronomy at the University of Wyoming.

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